Super Haavelmo: balanced and unbalanced budget theorems and the sraffian supermultiplier

Franklin Serrano, IE-UFRJ

Kaio Pimentel, IPPUR-UFRJ

Área 2: Crescimento econômico e distribuição de renda

Resumo

Este artigo estende a análise de Haavelmo (1945), que derivou o efeito multiplicador de uma expansão dos gastos publicos (mantendo o orçamento equilibrado) sobre a demanda agregada e o produto. Inicialmente generalizamos os resultados de Haavelmo mostrando que uma expansão fiscal pode ter resultados sobre a demanda e o produto até no caso de um superavit primario relativamente pequeno e estabelelcendo o principio geral de que o que interessa para a politica fiscal ser expansionista é que a propensão a gastar dos que são taxados seja mais baixa do que a do governo e daqueles que recebem trasnferências publicas. Mostramos também que a endogeneização do investimento induzido das empresas como uma propensão a investir faz o (super) multiplicador do orçamento equilibrado maior que um. Além disso, se esta propensão a investir muda ao longo do tempo e tende a ajustar a capacidade à demanda, de acordo com o modelo de crescimento liderado pela demanda do supermultplicador sraffiano, a carga tributária liquida que equilibra o orçamento tenderá a ser menor, quanto mais alta for a taxa de crescimento dos gastos do governo, mesmo na presença outros gastos autonomos privados.

Palavras-Chave: Teorema do Orçamento Equilibrado, Haavelmo, Supermultiplicador

Abstract

This paper extends the analysis of Haavelmo (1945), which derived the multiplier effect of a balanced budget expansion of public spending on aggregate demand and output. We first generalize Haavelmo´s results, showing that a fiscal expansion can have positive effects on demand and output even in the case of a relatively small primary surplus and establishing the general principle that what matters for fiscal policy to be expansionary is that the propensity to spend of those taxed should be lower that of the government and the recipients of government transfers. We also show that endogenizing business investment as a propensity to invest makes the traditional balanced budget multiplier to become greater than one. Moreover, if this propensity to invest changes over time and adjusts capacity to demand as in the sraffian supermultiplier demand led growth model, the net tax rate that balances the budget will tend to be lower the higher is the rate of growth of government spending, even in the presence of other private autonomous expenditures.

Keywords: Balanced Budget Theorem, Haavelmo, Supermultiplier

1.Introduction

This paper intends to reexamine and extend the analysis of Haavelmo (1945), which showed the multiplier effect of a balanced budget expansion of public spending on aggregate demand and output. We first generalize Haavelmo´s results showing that a fiscal expansion can have positive effects of demand and output even in the case of relatively small primary surplus and establish that what matters for fiscal policy to be expansionary is that the propensity to spend of those taxed should be lower that of the government and the recipients of government transfers. We also show that endogenizing business investment as a propensity to invest makes the traditional balanced budget multiplier to become greater than one. Moreover, if this propensity to invest changes over time and adjusts capacity to demand as in the Sraffian supermultiplier demand led growth model[1], the net tax rate (or "burden" for some) that balances the budget will tend to be lower the higher is the rate of growth of government spending, even in the presence of other private autonomous expenditures. Section 2 presents the two balanced budget theorems of Haavelmo that are of interest to us. Section 3 generalizes then. Section 4 introduces induced investment. Section 5 discusses the case in which capacity adjusts to demand via the sraffian supermultiplier. Section 6 contains brief final remarks.

2. **Fiscal policy with a balanced budget**

Haavelmo (1945) provided simple but rigorous proofs showing that, under certain assumptions discussed below, raising the level of public spending, even when this is fully financed by taxes, increases aggregate demand, output and income by the same amount.

With this work, Haavelmo sought to establish a theoretical basis to counter some common ideas at the time of his article, such as that a balanced budget would be neutral from the point of view of aggregate demand. According to this view, the effect of a balanced budget expansion of public expenditures would be zero, since it would only add to the aggregate demand public expenditures (G), of the same amount of the taxes (T) that it subtracts from the private sector. Some countered this by arguing that a possible expansionary impact of the balanced budget would depend a the redistribution of income through taxation, which could imply an increase in the aggregate marginal propensity to consume of the economy and, as a consequence, the size of the Keynesian multiplier.

What Haavelmo shows, as we shall see below from the presentation of the first theorem of the 1945 paper, is that the expansionary impact of the balanced budget is totally independent of the size of the marginal propensity to consume.

2.1 Autonomous Net Tax Revenue

To simplify the exposition, we will disregard the variation in inventories and assume a closed economy with idle labor and spare productive capacity. We also assume both for simplicity and ease of comparison with Haavelmo´s work that both the distribution of income and that of tax incidence are exogenously given, so that we can focus exclusively on the size and not the composition of taxation and public expenditures. For given levels of investment I and autonomous consumption Z we would have:

[1] In order to save space and focus on our specific analytical results, the paper does not enter in the debate of the merits and demerits of the sraffian (and now neokaleckian) supermultiplier growth model. For a recent debate on these issues , see the contributions to the special issue of Metroeconomica, may 2019 on "autonomous demand, capital utilization and growth" (Kurz & Salvadori (2019))

$$Y = Z + C + I + G \quad (1)$$

$$G = T \quad (2)$$

$$C = Z + c(Y - T) \quad (3)$$

Where Y means output and aggregate income, C is consumption, G is government spending, T means total taxes net of transfers and $0 < c < 1$ is the aggregate marginal propensity to consume. All variables are measured in real terms. Substituting (2) and (3) into (1) and developing:

$$Y = \frac{Z + I}{1 - c} + \frac{T(1 - c)}{1 - c} = \frac{Z + I}{1 - c} + T \quad (4)$$

Note that, for a given marginal propensity to consume, $0 < c < 1$, an equal increase in taxes and government expenditures is expansionary. The change in output and income ΔY is, in this case, equal to the change in taxes ΔT and public expenditure ΔG. Thus: $\Delta Y = \Delta T = \Delta G$. Therefore with autonomous (lump sum) taxation and government expenditures the balanced budget multiplier is equal to one. [2]

Note that as government spending, taxes and aggregate income increase by the same amount, the disposable income of the private sector (Y_D) in this case remains the same, independent of the size of the government expenditures and tax receipts:

$$Y - T = Y_D = \frac{Z + I}{1 - c} \quad (5)$$

Haavelmo points out that while disposable income regulates private consumption demand for goods and services, total expenditure is what determines the level of activity. Note that there is no change in private consumption, which is a function of disposable income, as aggregate income has increased but disposable income has not (because of the increase in taxes). Moreover, in this case, where it is assumed that c remains constant, an expansion of public expenditure fully financed by taxes has a positive effect on aggregate demand, output and income the same. Thus, fully financed public spending on taxes has an effect on aggregate income and on the level of output that is independent of the propensity to consume. Of course the expansionary effect of fiscal policy would be greater (and the multiplier greater than one) if taxes were raised by less than government spending as the ensuing government deficit would increase the disposable income of the private sector and induce more private consumption. But the point is that this is not needed for fiscal policy to increase aggregate demand and output.

2.2 A given net tax rate

[2] Note that the balanced budget theorem is basically independent of lags. If expenditures happen before taxes are collected there will be a temporary primary deficit and output will accordingly be temporarily above the equilibrium level. If taxes are collected before the expenditure there will be a temporary surplus and the level of output will first fall and then increase. But in the end will have increased by more than its initial fall anyway. We here assume the latter case in order not to have to discuss government deficits and their financing, something that does not interest us here, as we are concerned solely with balanced budgets (and primary surpluses).

Let us now see what happens if we take as given not the absolute amount of taxes but the net tax rate $t = \frac{T}{Y}$, making net tax revenue a function of aggregate income Y:

$$T = tY = G \tag{6}$$

By replacing (6) in (4) , the level of output for a given tax rate becomes:

$$Y = \frac{Z + I}{(1 - c)} . \frac{1}{(1 - t)} \tag{7}$$

In (7) we see that for given levels of Z and I and $0 < c < 1$, the greater the tax burden t, since $0 < t < 1$, the greater the level of output Y. And as this expansionary effect, depends only on the size of the increase of the tax rate, it is independent of the size marginal propensity to consume.

In the process of increasing the tax burden, the distribution of relative tax incidence can, of course, affect the distribution of disposable income, and thus have an impact on the aggregate marginal propensity to consume, as marginal propensities to consume of individual agents or classes are surely heterogeneous .[3] But as Haavelmo(1945) pointed out , the aim of his article was just to demonstrate that a balanced budget has a multiplier effect equal to the increase of government spending and taxes, in addition to any positive or negative effects arising from the redistribution of income caused by taxation.

In this context, this expansionary effect of government spending in fact occurs because the government's propensity to spend is supposed to be equal to unity, while $0 < c < 1$ is assumed for the private sector. Equation (8) below, also obtained by substituting (6) into (4) , but arranging it in a different way than Haavelmo did (our equation (7)) may perhaps make this more clear :

$$Y = \frac{Z + I}{1 - c - t(1 - c)} \tag{8}$$

Thus Haavelmo´s results do not depend on the specific size of the marginal propensity to consume of the private sector, apart from the fact that it should be lower than one, but his results are based on his assumption that the propensity to spend of the government is always equal to one. Therefore, the more general issue here is that the impact of higher (and equal) levels of taxation and government spending on aggregate demand and output is positive whenever the government's propensity to spend is greater than the marginal propensity to consume of the people that are being taxed. Thus, in the case under discussion, here in which the government's propensity to spend is equal to unity, the application this principle requires that :

$$t(1 - c) > 0 \tag{9}$$

[3] Our interest here concerns only the results of Theorems I and III of Haavelmo (1945). Theorem II deals with the form of the consumption function and is not relevant to us in this work because we are assuming consumption to be a linear function of disposable income. Theorem III , although not presented during this work, establishes the analytical conditions that would have to be satisfied so that the variations of the tax burden do not alter the distribution of income to the point of changing the aggregate marginal propensity to consume.

And this always holds whenever $0 < c < 1$

As with so many so-called "Keynesian" propositions , Kalecki (1937) had arrived at similar results a quite a few years earlier than Haavelmo. Kalecki (1937) argued that taxing capitalists and spending the money would add to an equal increase in aggregate effective demand, output and employment, as it would transfer to the government that would spend fully income that would have been in part saved by the capitalists. But he argued that taxing wages would not be expansionary at all, as increased government spending would be matched by an equal reduction in the consumption of workers. This result , , requires the assumption not only that workers propensity to consume is higher than that of capitalists, but that workers collectively really "spend what they get". that consume their wages entirely and do not save at all, not even small proportion of their income. Only under this particular assumption, explicitly made by Kalecki, the propensity to spend of the government is exactly the same as that of the workers, and condition (9) above does not hold.

3. A slightly more general unbalanced budget theorem

Let us now assume that, for political and institutional reasons the government, while still wanting to stimulate the economy, is forced not only to tax before spending, but also to obtain a certain target level of primary surplus. For the sake of comparison with the results of the previous section, we will look at two cases. In the first, the target primary surplus is given as a given fraction ρ of total autonomous tax revenue T. And in the second, the primary surplus target is set as a given proportion α of GDP and the net tax rate t is given.

3.1 A primary surplus as share of net tax revenues

In the first case, the fiscal policy will be represented by $G = T(1 - \rho)$. Replacing (2) by this new rule we get:

$$Y = \frac{Z + I}{1 - c} + \frac{T(1 - c)}{1 - c} = \frac{Z + I}{1 - c} + \frac{T(1 - c - \rho)}{1 - c} \tag{10}$$

Which can be rewritten as:

$$Y = \frac{Z + I}{1 - c} + T(1 - \frac{\rho}{1 - c}) \tag{11}$$

The second term on the right-hand side of equation (11) above shows that, even if a primary surplus has to be obtained, an increase in government expenditures financed by taxes can be expansionary, provided the primary surplus target as a fraction of tax revenue is not too big. Note that this depends on the primary surplus target being smaller than the marginal propensity to save of the private sector. This again shows that what matters is the difference being the government´s propensity to actually spend its tax revenues compared to the propensity to spend of those who are taxed. Note also that the size of this unbalanced budget theorem multiplier depends on the size of the marginal propensity to consume even with lump sum or autonomous net tax revenues. And the unbalanced budget multiplier, even when positive, will naturally be below one, as the government does not spend everything it taxes ($\rho > 0$).

3.2 Primary surplus as a share of output and income

Turning now to the second case, let us see how having to meet the primary surplus condition affects the results we have obtained in the previous section. A primary surplus

target reduces the government's own propensity to spend its tax revenues. If there is a primary surplus target of α per cent of output and income, we now have, for a given net tax rate :

$$G = (t - \alpha)Y \tag{12}$$

The demand determined level of output will then become:

$$Y = \frac{Z + I}{1 - c - t(1 - c) + \alpha} \tag{13}$$

Equation (13) above shows that any increase in the tax burden still increases the multiplier and the level of output, while any increase in the primary surplus target has a contractionary effect and reduces the multiplier and output. The term in the denominator that measures the government's contribution to the economy's marginal propensity to spend is now:

$$t(1 - c) - \alpha \tag{14}$$

In order for the government's overall contribution to the economy's propensity for spend to be positive, it is necessary that (17) is positive. Thus, the condition for the net impact of fiscal policy to be expansionary is:

$$\frac{\alpha}{t} < 1 - c \tag{15}$$

Thus, for the public sector contribute positively to the level of output, the government's propensity not to spend has to be lower than the private sector's marginal propensity to save. Similarly, we can also say that for the government contribution to be positive for the aggregate level of expenditures, as already mentioned, the government's propensity to spend must be greater than the private sector's propensity to spend the income that is being taxed.

Thus, it is possible that the impact of the public sector is positive even with the primary surplus target, provided that such target is not too large. If condition (15) above holds the positive net impact of fiscal policy will increase whenever the surplus target is reduced or whenever the tax burden is rising. In both cases, the economy´s aggregate marginal propensity to spend increases.

3.3 Impacts of different types of government spending and transfers

Up to now we have assumed that (apart from the primary surplus target) what the government actually spends has always an equal impact on aggregate demand. But different type of government expenditures and transfers, by their very nature, have very different impacts on demand. There are in principle three different cases. The first is when the government buys goods and services directly from the private sector (public investment in fixed capital and part of government consumption). In this case the direct impact of a unit of government expenditure on demand is clearly equal to one, as it is well known. Haavelmo (1945) implicitly assumed that all of G corresponds to this type of expenditure. On the other hand, we have government transfers of many types (including financial transfers if one considers interest paid on public debt, as we here do not). The

effect of those will clearly depend on the propensity to spend of those who receive these transfers, in our simple framework here represented by the marginal propensity to consume (we say to marginal propensity to spend instead of just to consume just because in a more general setting we could include the actual propensity to invest, of those who receive government subsidies specifically intended for that purpose, though it does not seem easy to find cases in which the effect of such subsidies is much different than zero). In the case of transfers, the impact of a unit of public expenditure on aggregate income would be equal to the marginal propensity to consume of those who receive the government transfers, and here the degree of progressivity of both taxation and government transfers is very important. But a proper discussion of this topic is way beyond our limited scope here, both because we are concerned only with the impact of levels, and not the structure, of government expenditure and taxation. Moreover for the sake of easy comparison with Haavelmo´s original results, here we assume that there is a single marginal propensity to consume (c).

But all this is well known. What to our knowledge has not been yet contemplated in the literature is the peculiar impact of the part of government spending on consumption through the direct provision of public services (such as health and education, for instance). When the government spends such services, the impact on the level of aggregate output as measured on national accounts is equal to its costs. In those costs, the goods and services bought from the private sector have already been discussed above. But in the case of the public sectors wage and salary bill, there is a difference. For in this case there is an extra impact. Let us take the example of a public university that does not charge fees to students and hires a new professor. Then the direct impact of this expenditure on gdp is that of an increase of the supply of public education of the same value as the new professor´s salary, something that gives the impression of the impact being no different of a purchase of a good of the same value from the private sector. However, university professors also spend their salaries, according to their own marginal propensity to consume. Thus, this type of government expenditure on personnel has an impact that is equivalent of being both an expenditure by the government *and* something equivalent to a transfer to the public employee (whose household belongs to the private sector) that will consume at least part of the earned wage. This is thus the most expansionary of all three types of government expenditure and transfers. Summing up, any government expenditure in goods and services (not only fixed capital goods) provided by the private sector implies an effective government propensity to spend on the economy equal to one, government transfers imply an impact of c, and expenditure on new public workers has an impact of $1 + c$. Thus, the overall relevant effective marginal propensity to spend of the government will, even in our simple framework, be some number between c and $1+ c$, depending on the composition of government expenditures and transfers. We shall call this average β.

This allows us a further generalization of the condition that the effective marginal propensity to spend of the government expenditures and transfers must be greater than that of the those being taxed in the private sector as:

$$\frac{\alpha}{\beta t} < 1 - c \tag{16}$$

Taking this factor into account, the level of output can be written:

$$Y = \frac{Z + I}{1 - c - t(\beta - c) + \alpha} \tag{17}$$

Adapting this to deal with distributive differences in taxation, transfers and marginal propensities to consume of different social groups is straightforward, but this is way beyond our limited scope here.

4. Balance budget expansions with induced investment

The cases represented in sections 2 and 3 so far have been restricted to the positive short run effects on the level of output of expansionary balanced (or more than balanced) budget increases in government spending. But in this short run Keynesian framework of Haavelmo, in which the level of investment is taken as given, each increase in the level of output brought about by higher taxes and spending imply an increase in the level of tax burden (tax as a share of gdp).

So we would be led to the conclusion that, while such type of fiscal policy could increase the level of effective demand, it would be less attractive politically as a means to generate a particular rate of growth of effective demand, not only because of the ever growing net tax rate but also because, for growth to be demand led over time, investment and capacity output must obviously grow, no matter how much spare capacity one assumes to be available in the beginning of the process.

But if we look over longer periods it is clear that business investment will not remain constant if demand is growing. If we bring in the longer run tendency of capacity to adjust to the trend of effective demand, the growth of final demand will lead to increases in induced investment.

When we take this into account, we see that a sustained increase in tax revenues and government spending at a certain rate will tend, after a while, to stimulate private investment to grow, as the degree of utilization of existing capacity increases. These new investments by their turn will cause through their own usual multiplier effects further increases in induced consumption and income. These increases in induced investment and consumption will no doubt increase tax revenues further and tend to counteract the tendency of the aggregate net tax rate to increase without limit.

4.1 The balanced budget (super) multiplier is greater than one

Let us then discuss these longer run issues by introducing one element at a time, starting with the assumption that aggregate private investment is induced by the level of output, following the sraffian supermultiplier demand led growth model [4]. Thus, there is initially an investment rate (or propensity to invest) that we will call h. Let us see how this new assumption change the results obtained so far. For the sake of simplicity and ease of comparison with Haavelmo´s results we shall from now on assume that the effective marginal propensity to spend of the government is equal to one and there is no need to generate a primary surplus (so, from now on $\alpha = 0$ and $\beta = 1$). The reintroduction of these elements would be straightforward and we think it would not lead to important qualitative changes in our results.

Recall that in equation (4) that presents the contribution of Haavelmo, the value of marginal propensity to consume was irrelevant, to the result that a balanced budget expansion has a multiplier equal to one,

[4]Serrano (1995), Serrano, Freitas & Bhering (2019).

With induced investment, we have that the equation that determines the level of output through the supermultiplier is:

$$Y = \frac{Z}{1-c-h} + \frac{T(1-c)}{1-c-h} \quad (18)$$

Or

$$Y = \frac{Z}{1-c-h} + \frac{T}{1-\frac{h}{1-c}} \quad (19)$$

In this new case, the balanced budget (super)multiplier is no longer equal to one, but to:

$$\frac{1-c}{1-c-h} \quad (20)$$

Or

$$\frac{1}{1-\frac{h}{1-c}} \quad (21)$$

Now, this balanced budget (super)multiplier is always greater than one provided that the usual condition of for the supermultiplier, that the aggregate marginal propensity to spend is lower than one, holds : $0 < c + h < 1$; And besides being greater than one, the size of this new balanced budget multiplier is larger, the larger is the propensity to invest and the marginal propensity to consume.

The reason for this is that a balanced budget increase in government expenditures increases aggregate demand and output. This increase leads to more induced investment. This increase will be higher, the higher the propensity to invest. This increase in induced investment will, by its turn generate induced consumption as workers hired in the capital goods industry receive their wages and spend part or all of them, this latter effect also being stronger, the higher is the aggregate marginal propensity to consume.

Note that because of these further increases in induced investment and consumption, here a balanced budget expansion of taxes and government spending, differently from Haavelmo, does also increase the disposable income of the private sector.

4.2 Growing with government expenditures as the only component of autonomous demand

Now we turn to the analysis of growth. In this first case, we shall assume that G , government spending is the only component of autonomous demand $Z = 0$ and the net tax rate t is adjusted in order to generate and keep balanced budget as government spending grows at a rate g_{gov} . Keeping the assumption that the propensity to invest $h = \frac{I}{Y}$ remains exogenously given, we see that now the level of output is determined by:

$$Y = \frac{G}{1 - c(1-t) - h} \tag{22}$$

To find the net tax burden that will balance the budget is done by simply solving for t the equality between the actual level of output and the level of output that would balance the budget ($G = tY$, $hence\ Y = \frac{G}{t}$):

$$Y = \frac{G}{1 - c(1-t) - h} = \frac{G}{t} \tag{23}$$

This gives us the tax rate that balances the budget t^* as :

$$t^* = \frac{1 - c - h}{1 - c} \tag{24}$$

Or:

$$t^* = 1 - \frac{h}{1 - c} \tag{25}$$

Note that the tax burden that balances the primary budget, in this case, is a negative function of the propensity to invest and the propensity to consume, and it is not a coincidence that it is, in this particular case, the inverse of the balance budget fiscal (super) multiplier. The higher the rate of investment of the economy and/or the greater the propensity to consume, the lower is the tax burden that balances the primary result.

We see then that mere existence of a positive marginal propensity to invest positive $h > 0$ implies (if government spending is the only source of autonomous demand) that the net tax burden no longer tends to unity if government spending and taxation grow together over time, since now (differently from the cases examined in sections 2 and 3) induced investment and induced consumption also tend will tend to grow at the same rate, and tax rate (equal to the share of government spending on gdp) will stabilize. The existence of h also implies that t^* is a negative function also of the propensity to consume (which determines the share of induced private consumption in income and output).[5]

4.2 Growing with both public and private autonomous demand

Now let's see how the results found are changed when we reintroduce the autonomous portion of private consumption (Z) and we assume that this expenditure grows at g_z.

The level of output is given by:

$$Y = \frac{G + Z}{1 - c(1-t) - h} \tag{26}$$

The tax rate that balances the budget is now:

$$t^* = \frac{1 - c - h}{1 - c + \frac{Z}{G}} \tag{27}$$

[5] Similar results are implicit (but not noted) in the demand led growth model driven by balanced budget expansion of government expenditures by Petri (2003).

In this case, the balanced budget tax burden will be a negative function of the investment rate h, the marginal propensity to consume c, and the evolution of the ratio between private autonomous spending and public expenditure $\frac{Z}{G}$.

Note any initial value of $\frac{Z}{G} > 0$ already allows us to have a lower balanced budget tax rate than the in previous case where, by hypothesis, there was no private sector autonomous demand. In fact, the existence of private autonomous expenditure reduces the share of government expenditures in output. But being g_{gov} the rate of growth of public spending and g_z the rate of growth of autonomous private spending, we can rewrite the above expression as:

$$t^* = \frac{1 - c - h}{1 - c + \frac{Z_{-1}(1 + g_z)}{G_{-1}(1 + g_{gov})}} \tag{28}$$

Here we see that any difference between g_z and g_{gov} implies that the ratio between private autonomous expenditure will change over time. Starting from a situation in which $\frac{Z}{G} > 0$, if $g_z > g_{gov}$ the tax rate required to balance the budget would tend to fall continuously, as private autonomous demand becomes an ever larger share of output and the growth rate of the economy as a whole approaches g_z. But this is not of interest to us here, since after all we are looking at ways in which fiscal policy and government spending could increase the rate of growth of the economy. So, in the relevant case in which government spending grows faster then private autonomous demand, $g_z < g_{gov}$, the tax burden necessary to balance the budget would tend to increase over time, as the share of government spending on output increases and the growth rate of the economy tends towards g_{gov}.

But even then, we can see that the net tax rate that balances the budget will be bounded from above and will always remain below one. Moreover, the tax burden that balances the budget will tend asymptotically to an upper limit equal to the value of t^* that obtains when there are no private autonomous expenditures, as the ratio $\frac{Z}{G}$ decreases over time.

Therefore, even if government spending grows faster than private autonomous spending the tax rate that balances the budget will eventually tend to stabilize, as autonomous private demand loses importance over time. The share of government in output and the tax rate will tend to stop increasing, and induced investment and induced consumption will also gradually tend to grow at a rate closer and closer to the rate of growth of government expenditures.

5. Growing government expenditures and the adjustment between productive capacity and the trend of demand

The assumption of a given propensity to invest is useful to illustrate how the rate of growth of autonomous demand can lead to long run growth of investment and the capital stock such that productive capacity and demand grow in line with each other, stabilizing the actual degree of capacity utilization. But this, by itself is insufficient to illustrate the tendency, imposed by competition, for the level of the capital stock and capacity to adjust to the level of aggregate effective demand. In the sraffian supermultiplier demand led growth model, the propensity to invest changes over time to allow the adjustment of capacity to demand and a tendency towards the normal degree of capacity utilization.

Thus, to complete our analysis, we shall now introduce this endogenous adjustment of the propensity to invest h and see how this affects the results of section 4.

For this purpose we shall make use of the simple induced investment function (taken from Serrano, Freitas & Bhering (2019), see also Garrido Moreira & Serrano (2019, Forthcoming), where the propensity to invest is a function of the expected trend rate of growth of demand:

$$I = v(g_e + \varphi)\, Y \tag{29}$$

Where $v = \frac{K}{Y_*}$ is the normal capital-output ratio, g_e is the expected rate of growth of the trend of demand, φ is the capital replacement or dropout rate.

The expected trend growth rate is given by:

$$g_e = g_{e-1} + x(g_{t-1} - g_{e-1}) \tag{30}$$

Where g_{e-1} is the expected trend growth rate in the previous period, x is a parameter which gives the reaction speed for the deviations between the actual growth rate of the previous period g_{t-1} and the expected trend growth rate for the previous period g_{e-1}.

Assuming that the model is dynamically stable, we know that the economy´s levels of both output and productive capacity will be always slowly tending towards fully adjusted positions and that the propensity to invest will tend towards its required level, and such level will be higher the higher the trend rate of growth of the economy, led by the growth of total autonomous demand as:

$$h = v(g + \varphi) \tag{31}$$

5.1 Fiscal expansion and the adjustment of capacity to demand

We now look at how this longer run tendency of capacity to adjust to demand modifies our results, starting with the case in which the only autonomous component of aggregate demand is (balanced budget) public expenditures that grow at the rate g_{gov}.

In this case, the fully adjusted levels of capacity output and output will tend to :

$$Y^* = Y = \frac{G}{1 - c(1-t) - v\left(g_{gov} + \varphi\right)} \tag{32}$$

And the net tax burden that balances the budget then becomes:

$$t^* = \frac{1 - c - v\left(g_{gov} + \varphi\right)}{1 - c} \tag{33}$$

Or :

$$t^* = 1 - \frac{v\left(g_{gov} + \varphi\right)}{1 - c} \tag{34}$$

As before the net tax burden that balances the budget remains lower than one, and tends to stabilize. But the interesting new result is that the level of the net tax burden required to balance the budget will be *lower* the faster government expenditures and tax revenue are growing. This follows directly from the fact that, in the sraffian supermultiplier model, a faster rate of growth of demand both requires and induces a permanently higher

propensity to invest. This leads to a permanently higher size of the supermultiplier and hence government expenditure becomes a lower share of total output[6].

5.2 Balanced budget fiscal expansion and the adjustment of capacity to demand with private and public autonomous demand

We turn now to discuss the more general where there is a positive level Z private sector autonomous expenditures that can grow at a rate g_z that may be different than the rate at which government spending is growing. We then get that the level of output tends to:

$$Y^* = Y = \frac{G + Z}{1 - c(1 - t) - v(g + \varphi)} \tag{35}$$

Where g is the average growth rate of total autonomous demand (public and private).

To define the tax burden that balances the primary outcome, we need to solve for t:

$$\frac{G + Z}{\left(1 - c(1 - t) - v(g + \varphi)\right)} = \frac{G}{t} \tag{36}$$

Giving us:

$$t^* = \frac{1 - c - v(g + \varphi)}{1 - c + \frac{Z_{-1}(1 + g_z)}{G_{-1}(1 + g_{gov})}} \tag{37}$$

Here again we see that any difference between g_z and g_{gov} implies that the ratio between private autonomous expenditure will change over time. So, in the relevant case in which government expenditures grow faster than private autonomous demand $g_z < g_{gov}$, the tax burden necessary to balance the budget would again initially tend to increase over time, as the share of government spending on output increases and the growth rate of the economy tends towards g_{gov}.

But even then, as in subsection 4.2 above, we can see that the net tax rate that balances the budget will be bounded from above and will always remain below one. Moreover, again, the tax burden that balances the budget in this case will also tend asymptotically to an upper limit equal to the value of t^* that obtains when there are no private autonomous expenditures, as the ratio $\frac{Z}{G}$ decreases over time.

When government spending grows faster than private autonomous spending, the tax rate that balances the budget will again tend to stabilize, as the share of government in output stabilizes when induced investment and induced consumption tend to grow at the same rate as government expenditures. All these effects remain when we make the propensity to invest endogenous. But now the endogeneity of the propensity to invest and the fact that it tends to become larger the larger the rate of growth of total autonomous demand, we have that the upper bound of the tax rate that balances the budget, which is that that would occur if government spending would be the sole source of autonomous demand, will itself be *lower* the *faster* is the rate of growth of public expenditures.

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[6] This result is also implicit (but not note nor discussed) in Allain's (2015) supermultiplier demand-led growth model driven only by the growth of balanced budget public expenditures.

We may therefore conclude that, even in the presence of a slower growing private autonomous component of demand, the faster the rate of growth of balanced budget government expenditure, the lower with tend to ultimately be the level of the required net tax rate or "burden" t^* .

Note that since this is an asymptotic result, the adjustment towards this permanently lower tax rate will take a lot of time, and probably never be reached before something else in the model changes. But at least the tendency of the tax rate to grow without limit under a balanced budget expansion will be checked much earlier by the slow adjustment of the investment share to the higher rate of growth of the economy, making induced investment growth faster than government spending, something that begins to counteract the tendency of the share of government spending in total aggregate demand and output to increase, when government expenditures grow faster than private autonomous demand.

6. Final Remarks

From our discussion above it becomes clear that governments can in principle always do something to stimulate effective demand both in the short and in the long run, even under fiscal rules that preclude incurring in primary deficits or require primary surpluses, as long as those are sufficiently small. Moreover, this applies also to countries that do not issue their own currency or are part of monetary union or to regions (even municipalities) within countries that do have sovereign currencies. If our results hold then the main difficulties will not be technical but instead will be related to the current widespread unwillingness to introduce progressive taxation in a sufficiently large scale.

References

ALLAIN, O (2014) 'Tackling the Instability of Growth: A Kaleckian-Harrodian Model with an Autonomous Expenditure Component.' Cambridge Journal of Economics. Rdoi:10.1093/cje/beu039.

FREITAS, F & SERRANO, F. (2015) Growth rate and level effects, the stability of the adjustment of capacity to demand and the Sraffian supermultiplier. Review of Political Economy, v. 27, n. 3, p. 258-281, 2015.

GARRIDO MOREIRA & SERRANO (2019 - *Forthcoming*) O debate envolvendo o efeito acelerador na controvérsia sobre o modelo do supermultiplicador sraffiano, Revista de Economia Contemporanea, *Forthcoming*.

HAAVELMO (1945). Multiplier effects of a balanced budget. *Econometrica: Journal of the Econometric Society,* 311-318.

KALECKI, M (1937) A Theory of Commodity, Income, and Capital Taxation. In Kalecki (1990).

KALECKI, M (1990) *Capitalism: Business Cycles and Full Employment. Vol. 1 of Collected Works of Michal Kalecki*, edited by J. Osiatynski. Oxford: Clarendon Press.

KURZ,H. & SALVADORI,N. (2019) "Introduction: Autonomous demand, capital utilization and economic growth", *Metroeconomica*, May.

PETRI, F. (2003) Should the theory of endogenous growth be based on Say's Law and the full employment of resources?. The Theory of Economic Growth: a 'Classical'

Perspective, Cheltenham, UK and Northampton, MA, USA: Edward Elgar, p. 139-60, 2003.

SERRANO, F. (1995) *The Sraffian Supermultiplier*, Unpublished Phd thesis, University of Cambridge, Cambridge, UK.

SERRANO, F & FREITAS, F. (2017) The Sraffian supermultiplier as an alternative closure for heterodox growth theory. European Journal of Economics and Economic Policies, v. 14, n. 1, p. 70-91, 2017.7

SERRANO, F.; FREITAS, F.; BHERING, G. The Trouble with Harrod: the fundamental instability of the warranted rate in the light of the Sraffian Supermultiplier. Metroeconomica, 2019.