Super Haavelmo: balanced and unbalanced budget theorems and the sraffian supermultiplier

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Abstract

This paper extends the analysis of Haavelmo (1945), which derived the multiplier effect of a balanced budget expansion of public spending on aggregate demand and output. We first generalize Haavelmo´s results showing that a fiscal expansion can have positive effects of demand and output even in the case of a relatively small primary surplus and establishing the general principle that what matters for fiscal policy to be expansionary is that the propensity to spend of those taxed should be lower that of the government and the recipients of government transfers. We also show that endogenizing business investment as a propensity to invest makes the traditional balanced budget multiplier to become greater than one. Moreover, if this propensity to invest changes over time and adjusts capacity to demand as in the sraffian supermultiplier demand led growth model, the net tax rate that balances the budget will tend to be lower the higher is the rate of growth of government spending, even in the presence of other private autonomous expenditures.

1.Introduction

This paper intends to reexamine and extend the analysis of Haavelmo (1945), which showed the multiplier effect of a balanced budget expansion of public spending on aggregate demand and output. We first generalize Haavelmo´s results showing that a fiscal expansion can have positive effects of demand and output even in the case of relatively small primary surplus and establish that what matters for fiscal policy to be expansionary is that the propensity to spend of those taxed should be lower that of the government and the recipients of government transfers. We also show that endogenizing business investment as a propensity to invest makes the traditional balanced budget multiplier to become greater than one. Moreover, if this propensity to invest changes over time and adjusts capacity to demand as in the Sraffian supermultiplier demand led growth model, the net tax rate (or "burden" for some) that balances the budget will tend to be lower the higher is the rate of growth of government spending, even in the

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presence of other private autonomous expenditures. Section 2 presents the two balanced budget theorems of Haavelmo that are of interest to us. Section 3 generalizes then. Section 4 introduces induced investment. Section 5 discusses the case in which capacity adjusts to demand via the sraffian supermultiplier. Section 6 contains brief final remarks.

2. **Fiscal policy with a balanced budget**

Haavelmo (1945) provided simple but rigorous proof to affirm that, under certain assumptions discussed below, raising the level of public spending, even if fully financed by taxes, increases aggregate demand, output and income.

With this work, Haavelmo sought to establish a theoretical basis to counter some common ideas at the time of his article, such as that a balanced budget would be neutral from the point of view of aggregate demand. According to this view, the effect of a balanced budget expansion of public expenditures would be zero, since it would only add to the aggregate demand public expenditures (G), of the same amount of the taxes (T) that it subtracts from the private sector. Some countered this by arguing that a possible expansionary impact of the balanced budget would depend a the redistribution of income through taxation, which could imply an increase in the aggregate marginal propensity to consume of the economy and, as a consequence, the multiplier of autonomous expenditures.

What Haavelmo shows, as we shall see below from the presentation of the first theorem of the 1945 paper, is that the expansionary impact of the balanced budget is totally independent of the value of the propensity to consume.

2.1 Autonomous Net Tax Revenue

To simplify the exposition, we will disregard the variation in inventories and assume a closed economy with idle labor and spare productive capacity. For given levels of investment I and autonomous consumption Z we would have:

$$Y = Z + C + I + G \tag{1}$$

$$G = T \tag{2}$$

$$C = Z + c(Y - T) \tag{3}$$

Where Y means output and aggregate income, C is consumption, G is government spending, T means total taxes net of transfers and $0 < c < 1$ is the aggregate marginal propensity to consume. All variables are measured in real terms. Substituting (2) and (3) into (1) and developing:

$$Y = \frac{Z + I}{1 - c} + \frac{T(1 - c)}{1 - c} = \frac{Z + I}{1 - c} + T \tag{4}$$

Note that, for a given propensity to consume, $0 < c < 1$, and increase in taxes, since it is totally spent, is expansionary. The change in output and income ΔY is, in this case, equal to the change in taxes ΔT and public expenditure ΔG. Thus: $\Delta Y = \Delta T = \Delta G$. Therefore with autonomous taxation and government expenditure the balanced budget multiplier is equal to one. [1]

Note in (4) that the disposable income of the private sector (Y_D) in this case remains the same after government intervention:

$$Y - T = Y_D = \frac{Z + I}{1 - c} \tag{5}$$

Haavelmo points out that while disposable income regulates private consumption demand for goods and services, total expenditure is what determines the level of activity. Note that there is no change in private consumption, which is a function of disposable income, as aggregate income has increased but disposable income has not (because of the increase in taxes). Moreover, in this case, where it is assumed that c remains constant, an expansion of public expenditure fully financed by taxes has a positive effect on aggregate demand, output and income the same. Thus, fully financed public spending on taxes has an effect on aggregate income and on the level of output that is independent of the propensity to consume.

2.2 A given net tax rate

[1] Note that the balanced budget theorem is basically independent of lags. If expenditures happen before taxes are collected there will be a temporary primary deficit and output will accordingly be temporarily above the equilibrium level. If taxes are collected before the expenditure there will be a temporary surplus and the level of output will first fall and then increase. But in the end will have increased by more than its initial fall anyway. We here assume the latter case (i.e., $G = T_1$) in order not to have to discuss government deficits and their financing, something that does not interest us here as we are concerned soley with balanced budgets (and primary surpluses).

Keeping all the other hypotheses by which we have arrived at the above result, let us now see what happens if we take as given not the absolute amount of taxes but the net tax rate $t = \frac{T}{Y}$, making tax collection as a function of aggregate income Y:

$$T = tY = G \tag{6}$$

The level of product assuming that the tax rate is given can then be described as[2]:

$$Y = \frac{Z + I}{(1 - c)(1 - t)} \tag{7}$$

Let us now analyze the impact on the level of output rate of an expansionary balanced budget fiscal policy financed by an increase in the tax burden t. The product level after a change in the tax burden (Δt) can be described as:

$$Y' = \frac{Z + I}{(1 - c)(1 - t - \Delta t)} \tag{8}$$

The growth rate of output (g) can be calculated by comparing the product levels before and after the change in the tax burden. That is, comparing $Y' e\ Y$. We would get:

$$1 + g = \frac{Y'}{Y} = \frac{\dfrac{Z + I}{(1 - c)(1 - t - \Delta t)}}{\dfrac{Z + I}{(1 - c)(1 - t)}}$$

By eliminating the terms $(Z + I)$ and $(1 - c)$, we arrive at:

$$1 + g = \frac{1 - t}{(1 - t - \Delta t)} \tag{9}$$

Note that the above growth rate (g) is also independent of the propensity to consume. Substituting (6) into (4) for data the levels of Z and I and $0 < c < 1$, the greater the tax burden t, since $0 < t < 1$, the greater the level of product Y:

$$Y = \frac{Y_D}{1 - t} = \frac{\left[\dfrac{Z + I}{1 - c}\right]}{[1 - t]} \tag{10}$$

[2] Here again, to avoid the possibility of even temporary primary deficits (see footnote 1) we assume that $G = (t.Y_{t-1})$

By the expression (10) above, where all tax collection is supposed to equal total public expenditure on the acquisition of goods and services over the same period, the product of full employment (Y^*) could be reached, raising the tax burden to the point where $Y = Y^*$.

In the process of increasing the tax burden, the distribution of taxes can affect the distribution of income, having an impact on the propensity to consume. If c is changed between Y and Y'changes are brought about in the presented result[3].

Thus, as the author himself points out, the aim of the article was to demonstrate that a balanced budget has a multiplier effect equal to unity, in addition to any positive or negative effects arising from the redistribution of income caused by taxation.

Assuming that changes in the tax burden do not change the distribution of income to the point of changing the propensity to consume, the expansionist impact of government spending always occurs because the government's propensity to spend is supposed to be equal to unity, while $0 < c < 1$ is assumed. The expression below, obtained by substituting (6) into (4), may make this clearer[4]:

$$Y = \frac{Z + I}{1 - c - t(1 - c)} \tag{11}$$

Generalizing the result of Haavelmo, from the result found in (11), in the case where public expenditures are financed by taxes previously collected, the impact of government spending on aggregate income is positive whenever the government's propensity to spend is greater that the propensity to spend of people taxed,. Thus, in the case discussed here in which the government's propensity to spend is equal to unity, in

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[3] Our interest in this text consists of the results of Theorems I and III of Haavelmo (1945). Theorem II deals with the form of the consumption function and is not relevant to us in this work because we are assuming a function of linear consumption, a function of disposable income. Theorem III, although not presented during this work, establishes the analytical conditions that would have to be satisfied so that the variations of the tax burden do not alter the distribution of income to the point of changing the propensity to consume.

[4] This expression is a modification in the form of presentation of the Theorem I of Haavelmo. Haavelmo (1945, p. 316) when performing the substitution of (6) in (4) uses the expression $Y = \frac{Z+I}{(1-c)(1-t)}$.

order for the government's contribution to be positive in aggregate income, it is necessary that:

$$-t(1-c) < 0 \tag{12}$$

This happens whenever $0 < c < 1$

With these results, Haavelmo also denies the common view that the expansionary effect of fiscal policy depends on a government deficit or even that increased spending would have to be financed by the issuance of currency or bonds to be expansionist. As seen above, these are not necessary attributes for an expansionary fiscal policy. Full employment could be achieved on a balanced budget and financed only by taxes previously collected.

3. A slightly more general unbalanced budget theorem

Let us now assume for some political and institutional reasons the government, while still wanting to stimulate the economy, is forced not only to tax before spend but also to obtain a certain target level of primary surplus. For the sake of comparison with the results of the previous section, we will look at two cases. First, the target primary surplus is given as a given fraction ρ of total autonomous tax revenue T. And in the second, the primary surplus target is set as a given proportion of GDP and the net tax rate t is given.

3.1 A primary surplus as share of net tax revenues

In the first case, the fiscal policy will be represented by $G = T(1-\rho)$. Replacing (2) by this new rule we get:

$$Y = \frac{Z+I}{1-c} + \frac{T(1-c)}{1-c} = \frac{Z+I}{1-c} + \frac{T(1-c-\rho)}{1-c} \tag{13}$$

Which can be rewritten as:

$$Y = \frac{Z+I}{1-c} + T(1 - \frac{\rho}{1-c}) \tag{14}$$

The second term on the right hand side of equation (14) above shows that, even if a primary surplus has to be obtained an increase in government expenditures financed by taxes can be expansionary, provided the primary surplus target as a fraction of tax revenue is not too big. Note that this depends on the primary surplus target being smaller than the marginal propensity to save of the private sector. This again shows that what matters is the difference being the government´s propensity to actually spend its tax revenues compared to the propensity to spend of those who are taxed. Note that the size of this unbalanced budget theorem multiplier depends on the size of the marginal propensity to consume even with lump sum or autonomous net tax revenues. And the unbalanced budget multiplier will naturally be below one as the government does not spend everything it taxes (ρ).

3.2 Primary surplus as a share of output and income

Turning now to the second case, let us see how having to meet the primary surplus condition impacts the results we have described in the previous section. A primary surplus targets reduces the government's propensity to spend its tax revenues. If there is a primary surplus target of α per cent of output and income, we now have, for a given net tax rate :

$$G = (t - \alpha)Y \tag{15}$$

The demand determined level of output will then become:

$$Y = \frac{Z + I}{1 - c - t(1 - c) + \alpha} \tag{16}$$

The above equation shows that in the closed economy any increase in the tax burden still increases the multiplier and the level of output, while any increase in the primary surplus target has a contractionary effect and reduces the multiplier and output. The term in the denominator that measures the government's contribution to the economy's marginal propensity to spend is now:

$$t(1 - c) - \alpha \tag{17}$$

In order for the government's overall contribution to the economy's propensity for spend to be positive, it is necessary that:

$$-t(1-c)+\alpha<0 \tag{18}$$

Thus, the condition for the net impact of fiscal policy to be expansionary is:

$$\frac{\alpha}{t}<1-c \tag{19}$$

Remembering that $1-c=s$

We have

$$\frac{\alpha}{t}<s \tag{20}$$

Thus, for the public sector contribute positively to the level of output, the government's propensity to save (or not to spend) has to be lower than the private sector's marginal propensity to save. Similarly, we can also say that for the government contribution to be positive for the aggregate level of expenditures, as already mentioned, the government's propensity to spend must be greater than the private sector's propensity to spend.

Thus, it is possible that the impact of the public sector is positive even with the primary surplus target, provided it is not too large. If condition (18) above holds net impact of the government will increase whenever the surplus target is reduced or whenever the tax burden is rising. In both cases, the economy´s marginal propensity to spend economy increases.

3.3 Impacts of different types of government spending and transfers

Up to now we have assumed that what the government actually spends (apart from the primary surplus target) has an equal impact on aggregate demand. But different type of government expenditures and transfers, by their very nature, have very different impacts on demand. There are in principle three different cases. The first is when the government buys goods and services directly from the private sector (public investment in fixed capital and part of government consumption). In this case the direct impact of a unit of government expenditure on demand is clearly equal to one, as it is well known. Haavelmo (1945) implicitly assumed that all of G corresponds to this type of expenditure. On the other hand, we have government transfers of many types (including financial transfers if one considers interest paid on public debt, as we here do not). The effect of those will clearly depend on the propensity to spend of those who receive these transfers, in our simple framework here represented by the marginal propensity to consume (we say to marginal propensity to spend instead of just to consume just because in a more general setting we could include the actual propensity to invest, of those who receive government subsidies specifically intended for that purpose, though it does not seem easy to find cases in which this is much different than zero). In the case of transfers, the impact of a unit of public expenditure on aggregate income would be equal to the marginal propensity to consume of those who receive the government transfers, and here the degree of progressivity of both taxation and government transfers is very important, but beyond our limited scope here where, for the sake of easy comparison with Haavelmo´s original results, there is a single marginal propensity to consume (c).

All this is well known. What to our knowledge has not been contemplated in the literature yet is the peculiar impact of the part of government spending on consumption through the direct provision of public services (such health and education). When the government spends such services, the impact on the level of aggregate output as measured on national accounts is equal to its costs. In those costs the goods and services bought from the private sector have already been discussed above. But in the case of the public sectors wage and salary bill there is a difference. For in this case there is an extra impact. Let us take the example of a public university that does not charge fees to students hires a new professor. Then the direct impact of this expenditure on gdp is that of an increase of the supply of public education of the same value as the new professor´s salary, what gives the impression of the impact being no different of a purchase of a good of the same value from the private sector. However, university professors do spend their salaries, according to their own marginal propensity to consume. Thus, this type of government expenditure on personnel has an impact that is equivalent of being both an expenditure by the government *and* something equivalent to a transfer to the public employee that will consume at least part of the earned wage. This is thus the most expansionary of all three types of government expenditure and transfers. Summing up, government expenditure in goods and services implies an effective government propensity to spend on the economy equal to one, government transfers imply an impact of c, and expenditure on new public workers has an impact of $1 + c$. Thus, the overall relevant effective marginal propensity to spend of the government will, even in our simple framework, be some average between c and 1+ c, depending on the composition of government expenditures and transfers. We shall call this average.

This allows us a generalization of the condition that the effective marginal propensity to spend of the government expenditures and transfers being greater than that of the those taxed in the private sector as:

$$\frac{\alpha}{\beta t} < 1 - c \tag{21}$$

Taking this factor into account, the level of output can be written:

$$Y = \frac{Z + I}{1 - c - t(\beta - c) + \alpha} \tag{22}$$

Adapting this to deal with distributive differences in taxation, transfers and marginal propensities to consume is straightforward, but beyond our limited scope here.

4. Balance budget expansions with induced investment

The cases represented in sections 2 and 3 so far have been restricted to the positive short run effects on the level of output of expansionary balanced (or overbalanced) budget increases in government spending. But in this short run Keynesian framework of Haavelmo, in which the level of investment is taken as given, each increase in the level of output brought about by higher taxes and spending imply an increase in the level of tax burden (tax as a share of gdp).

So we would be led to the conclusion that, while such type of fiscal policy could increase the level of effective demand, it would be less attractive politically as a means to generate a particular rate of growth of effective demand, not only because of the ever growing net tax rate but also because for growth to be demand led over time investment and capacity output must obviously grow, no matter how much spare capacity one assumes to be available in the beginning of the process.

But if we look over longer periods business investment will not remain constant. If we bring in the longer run tendency of capacity to adjust to the trend of effective demand, the growth of final demand will lead to increases in induced investment.

When we take this into account, we see that a sustained increase in tax revenues and government spending at a certain rate will tend, after a while to stimulate private investment to grow, as the degree of utilization of existing capacity increases. These new investments by their turn will cause through their own usual multiplier effects further increases in induced consumption and income. These increases in induced investment and consumption will no doubt increase tax revenues further and tend to counteract the tendency of the aggregate tax rate to increase without limit.

4.1 The balanced budget multiplier is greater than one

Let us then discuss these longer run issues by introducing one element at a time, starting with the assumption that aggregate private investment is induced by the level of output [5]. Thus there is initially an investment rate (or marginal propensity to invest) that we will call h. Let us see how this changes the results obtained so far. For the sake of simplicity and ease of comparison with Haavelmo´s results we shall from now on assume that the effective marginal propensity to spend of the government is equal to one and there is no need to generate a primary surplus (so , from now on $\alpha = 0$ and $\beta = 1$). The reintroduction of these elements would be straightforward and would not imply any important qualitative change in our results.

Recall that in equation (4) that presents the contribution of Haavelmo, the value of marginal propensity to consume was irrelevant to the budget theorem balanced,

[5] Kalecki (1944) assumed that the rate of profit realized was a central determinant for investment and therefore advocated that changes in the tax burden be accompanied by changes in the tax structure so as not to reduce the rate of profit for new investments. These changes consisted of increasing taxation of accumulated wealth and / or taking the net and gross investment for the calculation of income tax. We do not share Kalecki's initial assumption about the relevance of the realized rate of profit or even, as in other neo-Kaleckian versions, from the normal profit rate to the dynamics of aggregate investment. In capitalist economies, the productive capacity aims to meet the effective demand, demand that pays the minimum profitability accepted by the capitalists. If the normal rate of profit is above the interest rate, competition causes companies to seek to meet effective demand otherwise they will lose market share. Thus, the rate of profit must be seen as a constraint (in case this is below the interest rate there is no reason for capitalists to invest) and not as a central determinant, since what determines the expansion of productive capacity, is the growth of effective demand. For more on this vision of growth led by demand, see Serramo (1995), Serrano and Freitas (2017) and Freitas and Serrano (2015).

provided it was considered given, that is, it was the same in the periods before and after taxation[6].

With induced investment we have that the equation that determines the level of output through the supermultiplier is:

$$Y = \frac{Z}{1-c-h} + \frac{T(1-c)}{1-c-h} \qquad (23)$$

Or

$$Y = \frac{Z}{1-c-h} + \frac{T}{1-\frac{h}{1-c}} \qquad (24)$$

In this new case, the multiplier of the balanced budget is no longer unitary, but equal to:

$$\frac{1-c}{1-c-h} \qquad (25)$$

Or

$$\frac{1}{1-\frac{h}{1-c}} \qquad (26)$$

This balanced budget multiplier is always greater than one provided that the usual condition of a for the supermultiplier that the aggregate marginal propensity to spend is lower than one : $0 < c + h < 1$; Besides being greater than one the size of this new balanced budget multiplier equilibrium is larger the larger is the propensity to invest and the marginal propensity to consume.

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[6] The equation (4) was: $Y = \frac{Z+I}{1-c} + \frac{T(1-c)}{1-c} = \frac{Z+I}{1-c} + T$. The balanced budget multiplier was unitary because: $\frac{1-c}{1-c} = 1$

The reason for this is that a balanced budget increase in government expenditures increases aggregate demand and output. This increase leads to more induced investment. This increase will be higher, the higher the propensity to invest. This increase in induced investment will, by its turn generate induced consumption as workers hired in the capital goods industry receive their wages and spend part or all of them , this latter effect also being stronger the higher is the aggregate marginal propensity to consume.

Note that because of these further increases in induced investment and consumption , here a balanced budget expansion of taxes and government spending , differently from Haavelmo, does also increase the disposable income of the private sector.

4.2 Growing with government expenditures as the only autonomous demand component

Now we turn to the analysis of growth. In this first case, we shall assume that G government spending is the only component of autonomous demand $Z = 0$ and the net tax rate t is adjusted in order to generate and keep balanced budget as government spending grows at a rate g_{gov} . Keeping the assumption that the propensity to invest $h = \frac{I}{Y}$ remains exogenously given, we that now the level of output is determined by:

$$Y = \frac{G}{1 - c(1 - t) - h} \qquad (27)$$

To find out the net tax burden that will balance the budget is done by simply solve for t the equality between the actual level of output and the level of output that would balance the budget ($G = tY$, $hence\ Y = \frac{G}{t}$):

$$Y = \frac{G}{1 - c(1 - t) - h} = \frac{G}{t} \qquad (28)$$

This gives us the tax rate that balances the budget[7] t^* as :

[7] The most well-known investigation to arrive at this expression for a tax rate that balances the budget, although incomplete, is in Ackley (1961, p. 344)

$$t^* = \frac{1 - c - h}{1 - c} \tag{29}$$

Or:

$$t^* = 1 - \frac{h}{1 - c} \tag{30}$$

Note that the tax burden that balances the primary budget, in this case, is a negative function of the propensity to invest and the propensity to consume, and it is not a coincidence that it is, in this particular case, the inverse of the balance budget multiplier. The higher the rate of investment of the economy and/or the greater the propensity to consume, the lower is the tax burden that balances the primary result.

We see then that mere existence of a marginal propensity to invest positive $h > 0$ implies (if government spending is the only source of autonomous demand) that the net tax burden no longer tends to unity if government spending grows over time, since now (differently from the cases examined in sections 2 and 3) induced investment and induced consumption also tend will tend to grow at the same rate . The existence of h also implies that t^* is a negative function also of the propensity to consume (which fixes the share of induced private consumption in income and output).[8]

4.2 Growing with both public and private autonomous demand

Now let's see how the results found are changed when we reintroduce the autonomous portion of private consumption (Z) and we assume that this expenditure grows at g_z. Examples of spending on these expenditures may be consumption financed by credit, residential investment or capitalist consumption.

[8] Similar results are implicit (but not noted) in the demand led growth model driven by balanced budget expansion of government expenditures by Petri (2003).

We are following the concept of autonomous/unproductive expenditures of Serrano (1995)[9]. These are the expenditures that do not directly generate productive capacity for the private sector and are not funded by the economy's wage payroll.

However, note that, for the purposes of this paper, it is important the difference between autonomous public and private expenditure.

Assuming G and Z autonomous, the product level is given by:

$$Y = \frac{G + Z}{1 - c(1 - t) - h} \tag{31}$$

In the above case, the level of tax burden that goes on to balance the result tax is:

$$t^* = \frac{1 - c - h}{1 - c + \frac{Z}{G}} \tag{32}$$

In this case, the tax burden that balances the tax result will be a negative function of the investment rate h, the marginal propensity to consume c, and the evolution of the ratio between private autonomous spending and public expenditure $\frac{Z}{G}$.

Note that for any initial value of $\frac{Z}{G} > 0$ already allows us to have a lower balanced budget tax rate than the in previous case where, by hypothesis, there was no private sector autonomous demand. In fact, the existence of private autonomous expenditure reduces the share of government expenditures in output. But being g_{gov} the rate of growth of public spending and g_z the rate of growth of autonomous private spending, we can rewrite the above expression as:

[9] On the various types of expenditures that make up autonomous spending, see Serrano (1995, p. 15-16): "The include the types of expenditure that should be considered autonomous according to our criterion consumption of capitalists; the discretionary consumption of richer workers that have some accumulated wealth and access to credit; residential 'investment' by households; discretionary expenditures (that are sometimes classified as 'investment' and sometimes as 'intermediate consumption' in official statistics) that do not include the purchase of produced means of production such as consultancy services, research & development, publicity, executive jets, etc. (...); government expenditure (both consumption and investment); and total exports (both of consumption and capital goods since the latter do not create capacity within the domestic economy). "

$$t^* = \frac{1 - c - h}{1 - c + \frac{Z_{-1}(1 + g_z)}{G_{-1}(1 + g_{gov})}} \tag{33}$$

Here we see that any difference between g_z and g_{gov} implies that the ratio between private autonomous expenditure will change over time. Starting from a situation in which $\frac{Z}{G} > 0$, if $g_z > g_{gov}$ the tax rate required to balance the primary result would tend to faal continuously as private autonomous demand becomes an ever larger share of output and the growth rate of the economy as a whole approaches g_z. But this is not of interest to us, since after all we are looking at ways in which fiscal policy and government spending could increase the rate of growth of the economy. So, in the relevant case $g_z < g_{gov}$, the tax burden necessary to balance the budget would tend to increase over time, as the share of government spending on output increases and the growth rate of the economy tends towards g_{gov} .

But even then we can see that the net tax rate that balances the budget will be bounded from above and will always remain below one. Moreover, the tax burden that balances the budget will tend asymptotically to an upper limit equal to the value of t^* that obtains when there are no private autonomous expenditures, as the ratio $\frac{Z}{G}$ decreases over time.

Therefore, even if government spending grows faster than private autonomous spending the tax rate that balances the budget will tend to stabilize, as the share of government in output stabilizes when induced investment and induced consumption tend to grow at the same rate as government expenditures.

5. Growing government expenditures and the adjustment between productive capacity and the trend of demand

The assumption of a given propensity to invest is useful to illustrate how the rate of growth of autonomous demand can lead to long run growth of investment and the capital stock such that productive capacity and demand grow in line with each other, stabilizing the actual degree of capacity utilization. But this, by itself is insufficient to illustrate the tendency, imposed by competition, for the level of the capital stock and capacity to adjust to the level of aggregate effective demand. In the sraffian supermultiplier demand led growth model the propensity to invest changes over time to allow the adjustment of capacity to demand and a tendency towards the normal degree of capacity utilization. Thus, to complete our analysis, we shall now introduce this endogenous adjustment of the propensity to invest h and see how this affects the results of section 4.

For this purpose we shall make use of the simple induced investment function (taken from Serrano, Freitas & Bhering (2019), see also Garrido Moreira & Serrano (Forthcoming), where the propensity to invest is a function of the expected trend rate of growth of demand:

$$I = v(g_e + \varphi)\, Y \tag{34}$$

Where $v = \frac{K}{Y*}$ is the normal capital-output ratio, g_e is the expected rate of growth of the trend of demand, φ is the depreciation dropout rate.

The expected growth rate is given by:

$$g_e = g_{e-1} + x(g_{t-1} - g_{e-1}) \tag{35}$$

Where g_{e-1} is the expected growth rate in the previous period, x is a parameter which gives the reaction speed for the deviations between the growth rate of the previous period g_{t-1} and the expected growth rate for the previous period g_{e-1}.

Note that now the investment function has an element g_e which contains an prospective element (g_{e-1}) that attempts to predict future growth rate and a corrective element $(g_{t-1} - g_{e-1})$which corrects the prediction errors.

Assuming that the model is dynamically stable, we know that the economy´s levels of both output and productive capacity will be always tending to a fully adjusted positions and that the propensity to invest will tend towards its the required level, and will be higher the higher the trend rate of growth of the economy, led by the growth of total autonomous demand as:

$$h = v(g + \varphi) \tag{36}$$

5.1 Fiscal expansion and the adjustment of capacity to demand

Let's see how this changes the results of the previous section, starting with the case in which the only autonomous component of aggregate demand is (balanced budget) public expenditures that grow at the rate g_{gov}.

In this case, the fully adjusted levels of capacity output and output will tend to :

$$Y^* = Y = \frac{G}{1 - c(1-t) - v\left(g_{gov} + \varphi\right)} \tag{37}$$

And the net tax burden that balances the budget then becomes:

$$t^* = \frac{1 - c - v\left(g_{gov} + \varphi\right)}{1 - c} \tag{38}$$

Or :

$$t^* = 1 - \frac{v\left(g_{gov} + \varphi\right)}{1 - c} \tag{39}$$

As before the net tax burden that balances the budget remains lower than one and tends to stabilize. But the interesting new result is that the level of the net tax burden required to balance the budget will be lower the faster government expenditures and tax revenue are growing. This follows directly from the fact that, in the sraffian supermultiplier model, a faster rate of growth of demand both requires and induces a permanently higher propensity to invest. This leads to a permanently higher size of the supermultiplier and this makes government expenditure a lower share of total output[10].

5.2 Balanced budget fiscal expansion and the adjustment of capacity to demand with private autonomous demand

We now discuss the more general where there is a positive level Z private sector autonomous expenditures that can grow at a rate g_z that may be different than the rate at which government spending is growing. We then get that the level of output tends to:

[10] This result is implicit (but not discussed) in Allain's (2015) supermultiplier demand-led growth model driven only by the growth of balanced budget public expenditures.

$$Y^* = Y = \frac{G+Z}{1-c(1-t)-v(g+\varphi)} \tag{40}$$

Where g is the average growth rate of total autonomous demand (public and private).

To define the tax burden that balances the primary outcome, we need to solve for t:

$$\frac{G+Z}{\left(1-c(1-t)-v(g+\varphi)\right)} = \frac{G}{t} \tag{41}$$

Giving us:

$$t^* = \frac{1-c-v(g+\varphi)}{1-c+\frac{Z_{-1}(1+g_z)}{G_{-1}(1+g_{gov})}} \tag{42}$$

Here again we see that any difference between g_z and g_{gov} implies that the ratio between private autonomous expenditure will change over time. So, in the relevant case in which government expenditures grow faster than private autonomous demand $g_z < g_{gov}$, the tax burden necessary to balance the budget would again initially tend to increase over time, as the share of government spending on output increases and the growth rate of the economy tends towards g_{gov} .

But even then , as in subsection 5.2 above, we can see that the net tax rate that balances the budget will be bounded from above and will always remain below one. Moreover, again, the tax burden that balances the budget in this case will also tend asymptotically to an upper limit equal to the value of t^* that obtains when there are no private autonomous expenditures , as the ratio $\frac{Z}{G}$ decreases over time.

When government spending grows faster than private autonomous spending the tax rate that balances the budget will again tend to stabilize, as the share of government in output stabilizes when induced investment and induced consumption tend to grow at the same rate as government expenditures. All these effects remain when we make the propensity to invest endogenous. But now the endogeneity of the propensity to invest and the fact that it tends to become larger the larger the rate of growth of total autonomous demand, we have that the upper bound of the tax rate that balances the budget, which is that that would occur if government spending would be the sole source of autonomous demand, will itself be *lower* the *faster* is the rate of growth of public expenditures.

We may therefore conclude that, even in the presence of a slower growing private autonomous component of demand, the faster the rate of growth of balanced budget government expenditure the lower with tend to ultimately be the level of the required net tax rate or "burden" t^* .

7. Final Remarks

From our discussion above it becomes clear that governments can in principle always do something to stimulate effective demand both in the short and in the long run, even under fiscal rules that preclude incurring in primary deficits or require primary surpluses, as long as those are sufficiently small. This applies to countries that do not issue their own currency or are part of monetary union or to regions (even municipalities) within countries that do have sovereign currencies. If our results hold then the main difficulties will not be technical but will be related to the modern widespread unwillingness to introduce progressive taxation in a sufficiently large scale.

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